Numerical Methods and the Rabinovich-Fabrikant Equations

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Chaos

- A property of some systems which change over time.
- Appears random but is not
- Important real-world applications:
  - Weather
  - Fluid dynamics
  - Electrical circuits
  - Chemical reactions

Runge-Kutta method

$$
\begin{aligned}
\mathbf{x}_{n+1} - \mathbf{x}_n &= \frac{h}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4) \\
\mathbf{k}_1 &= \mathbf{f}(\mathbf{x}_n, \mathbf{x}_n) \\
\mathbf{k}_2 &= \mathbf{f}(\mathbf{x}_n + \frac{1}{2}h, \mathbf{x}_n + \frac{1}{2}h\mathbf{k}_1) \\
\mathbf{k}_3 &= \mathbf{f}(\mathbf{x}_n + \frac{1}{2}h, \mathbf{x}_n + \frac{1}{2}h\mathbf{k}_2) \\
\mathbf{k}_4 &= \mathbf{f}(\mathbf{x}_n + h, \mathbf{x}_n + h\mathbf{k}_3)
\end{aligned}
$$

- Popular
- Well-documented [1, 2]
- Easy to implement

Local Iterative Linearization method

$$
\begin{aligned}
\mathbf{x}_k = {} & 2\mathbf{x}_{k-1} - \frac{8}{5}\mathbf{x}_{k-2} + \frac{26}{35}\mathbf{x}_{k-3} - \frac{1}{7}\mathbf{x}_{k-4} + \\
& \frac{h}{12600}(6463\mathbf{f}_k - 2092\mathbf{f}_{k-1} + 2298\mathbf{f}_{k-2} \\
& -1132\mathbf{f}_{k-3} + 223\mathbf{f}_{k-4})
\end{aligned}
$$

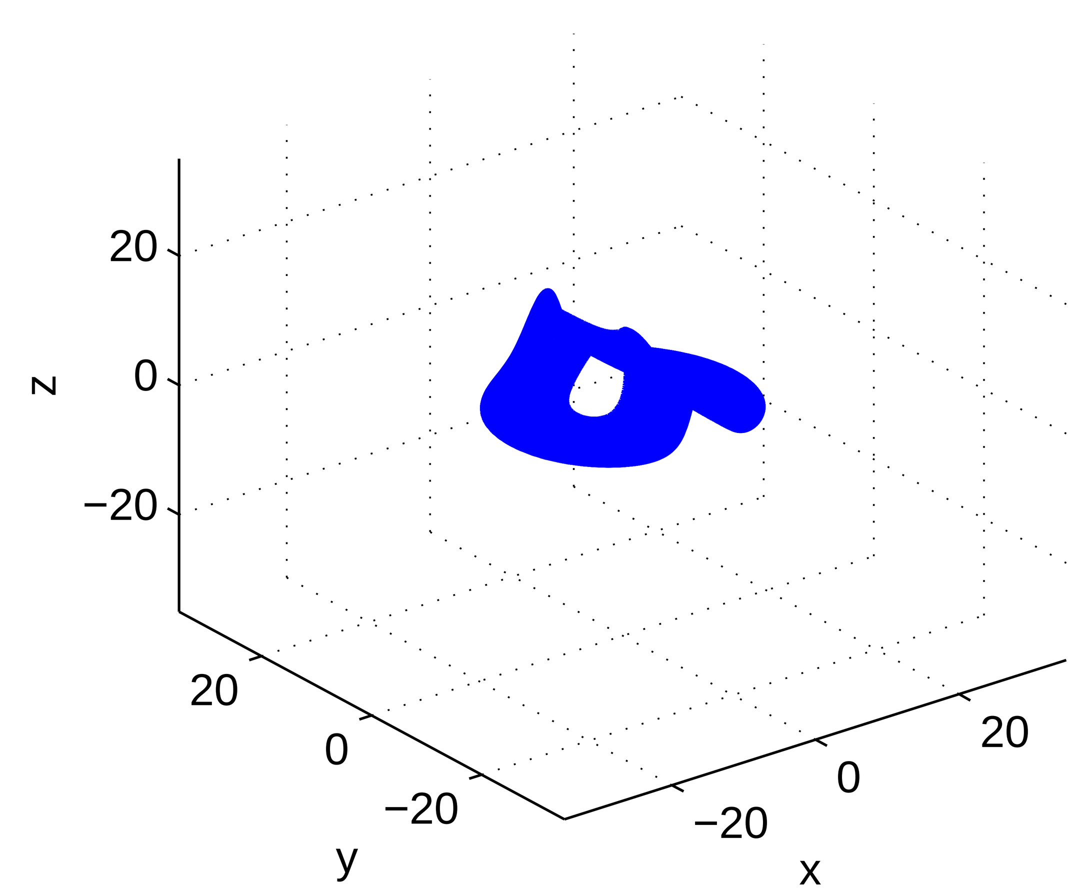
- Obscure
- Relatively few publications [3, 4, 5]
- More difficult to implement

The Rabinovich-Fabrikant equations

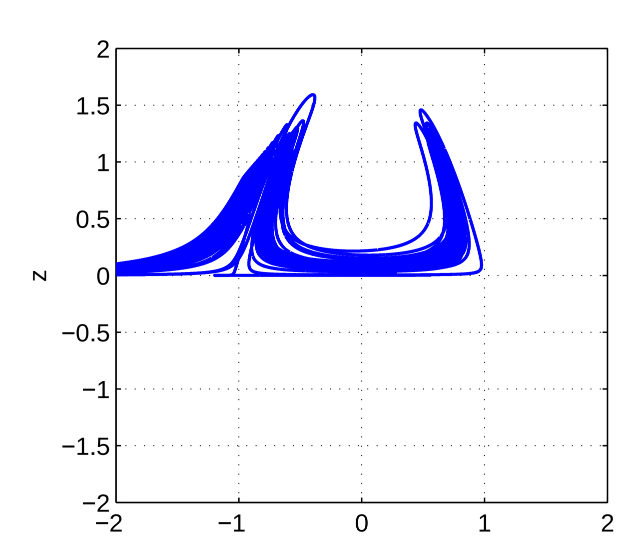
$$
\begin{aligned}
x' &= y(z - 1 + y^2) + ax \\
y' &= x(3z + 1 - x^2) + ay \\
z' &= -2z(b + xy)
\end{aligned}
$$

where $a, b > 0$ [6].

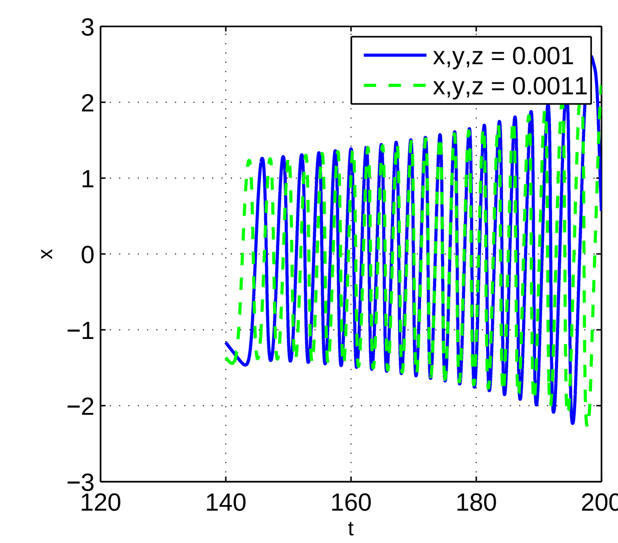
Local iterative linearization: $a = 0.1, b = 0.05$



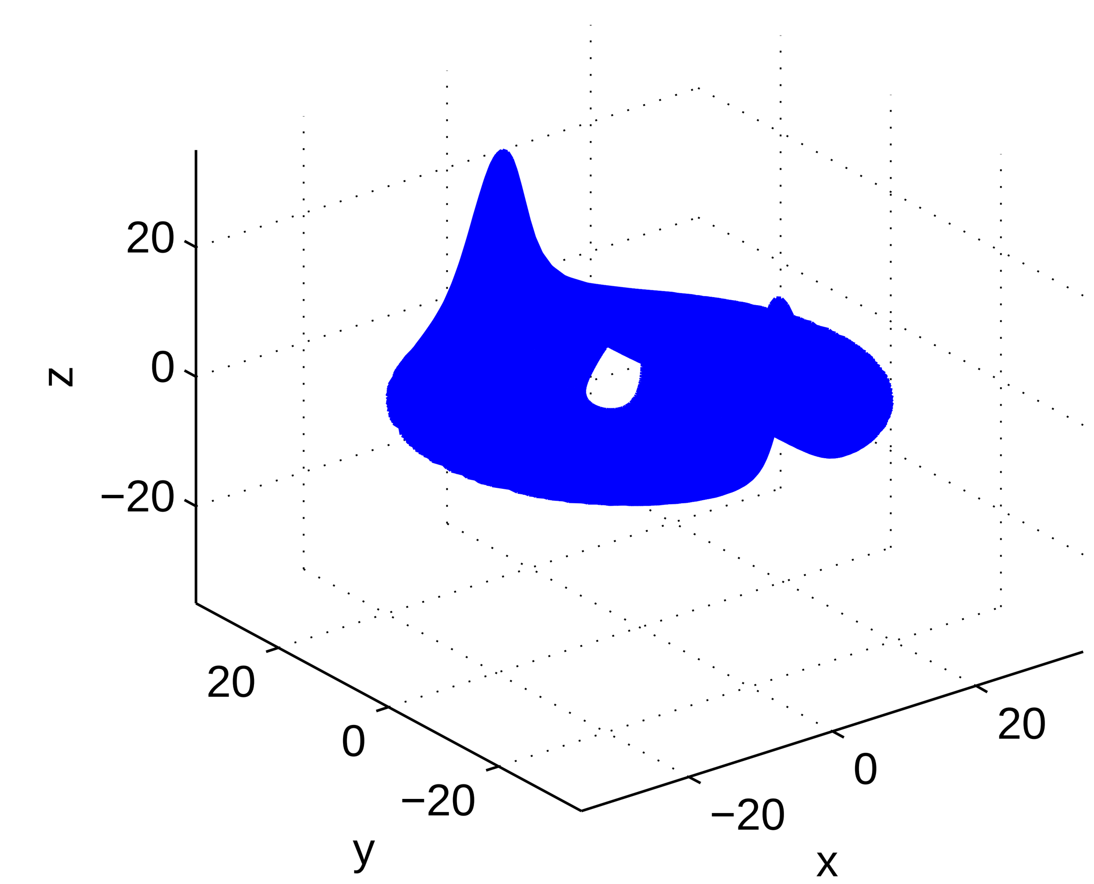
LIL: $a = 0.1, b = 0.14$



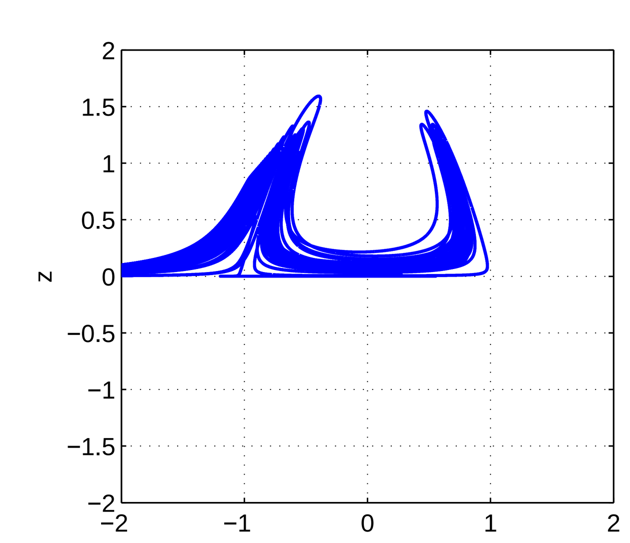
Sensitive dependence on initial conditions



Runge-Kutta: $a = 0.1, b = 0.05$



Runge-Kutta: $a = 0.1, b = 0.14$



Conclusions

- The Rabinovich-Fabrikant equations are difficult to study numerically.
- Local Iterative Linearization and Runge-Kutta methods are comparable for these equations
- Robust numerical methods for the approximation of chaotic dynamical systems do not exist
- Research directions for developing more effective numerical methods include:
  - Extended precision computing
  - Experimentation with different methods
  - Investing in new computer architectures that efficiently perform extended precision floating-point arithmetic
  - Developing higher-order (more accurate) algorithms for existing computer architecture
- Better-suited methods discovered in the future can be applied to other chaotic equations
- Chaotic systems of differential equations that model important real world events do not have analytical solutions. It is imperative that reliable numerical methods be developed that give approximate solutions that can be used with confidence.

References

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[3] Luo, Xiaodong, Michael Small, M.F. Danca, Guanrong Chen. On a dynamical system with multiple chaotic attractors. *Int. J. Bif. Chaos.,* **17, 9** (2007) 3235-3251

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[5] Danca, M.F., Guanrong Chen. Bifurcation and chaos in a complex model of dissipative medium. *Int. J. Bif. Chaos,* **14,10** (2004) 3409-3447.

[6] Rabinovich, M.I., A.L. Fabrikant. Stochastic self-modulation of waves in nonequilibrium media. *Zh. Eksp. Teor. Fiz. (Sov),* **77** (1977) 617-629.

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