Radial Basis Function Approximation Methods

Maggie E. Chenoweth
Department of Mathematics, Marshall University

Introduction

• RBFs first studied by Roland Hardy - 1968
  \[ s(x) = \sum_{j=1}^{N} \lambda_j \phi(\|x - x_j\|, \epsilon) \]
• Allow for scattered data sites to be easily used in computations
• Used to represent topographical surfaces and other 3-D shapes
  – Facial recognition
  – Ocean floor mapping
  – Medical applications

The most popular RBF that is used in applications today is the Multiquadric (MQ)
  \[ \phi(r) = \sqrt{1 + \epsilon^2 r^2} = (1 + \epsilon^2 r^2)^{1/2} \]
– Properties of the MQ are well-known [2]

A related RBF with properties not as well-known is the Generalized Multiquadric (GMQ)
  \[ \phi(r) = (1 + \epsilon^2 r^2)^{\beta} \]
– Researchers have recently suggested, but not proven, that the GMQ has desirable properties for \( \beta \) with non-half-integer powers.

GMQ - Suggested Values for \( \beta \)

• Wang and Liu [3]: \( \beta = 1.03 \)
• Xaio and McCarthy [4]: \( \beta = 1.99 \)
• Kansa [1]: \( \beta = 5/2 \)

Which value is best?

Two test functions:
  \[ f(x) = e^{x^3} + \cos(2x) \]  (1)
  \[ f(x) = x^4 + 3x^2 - x - 2 \]  (2)

Results:
• No optimal shape parameter was found for the six values of \( \beta \)
• As beta increases the shape parameters increase
• \( \beta = 2.5 \) provided the best results for the second function, but \( \beta = 1.99 \) provided the best results for the first function
• \( \beta \) is problem dependent

Extended Precision

RBF methods can be accurately and efficiently evaluated using extended precision floating point arithmetic
• Implemented using computer software instead of hardware
• Takes more computation time
• Provides better accuracy

<table>
<thead>
<tr>
<th>Comparison of Floating Point Types</th>
<th>type</th>
<th>bits</th>
<th>p</th>
<th>dps</th>
<th>exec time</th>
</tr>
</thead>
<tbody>
<tr>
<td>double</td>
<td>64</td>
<td>53</td>
<td>16</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>double-double</td>
<td>128</td>
<td>106</td>
<td>32</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>quad-double</td>
<td>256</td>
<td>212</td>
<td>64</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

\( p \) = bits in decimal section
\( dps \) = accurate decimal places

Using the following function
  \[ f(x) = e^{\sin(x)} \]  (3)

we found that the 256 quad-double appears to be best

Summary

• Optimal values for \( \beta \) are problem dependent
• Extended precision adds accuracy to RBF methods
  – Multi-core architecture of modern computers
  – Domain decomposition approaches

Acknowledgements

Special thanks to my research advisor, Dr. Scott Sarra.
Support for this project was provided in part by the NASA West Virginia Space Grant Undergraduate Fellowship Program.

References